CALCULUS WRITING PROMPTS

Jason D. Johnson
Zayed University, Department of Mathematics and Statistics, P.O. Box 19282, Dubai – UAE
Email: jason.johnson@zu.ac.ae

Carla A. Rudder
Rabdan Academy, Department of Mathematics, Al Dhafeer Street, Abu Dhabi – UAE
Email: cruder@ra.ac.ae

Abstract: In this paper, a study was designed to explore mathematics majors engaged in Calculus writing prompts. The paper also provides mathematics instructors benefits for instruction and students for the importance of writing to make sense of mathematics. An analysis for both the derivative and integral writing prompts was completed. The results indicate that mathematics majors were able to solve derivative and integral problems, however, some were unable to articulate their mathematical thinking and the thinking of others.

Key Words: Calculus, writing prompts, higher education, mathematics

"You know what I mean….I just can't explain it!"

Introduction

The above response was expressed by a mathematics major who participated in the study. University instructors have experienced leaners who are unable to articulate their mathematical understanding verbally and/or in written form. Why are some learners unable to convey their mathematical thinking in writing? Many learners view writing as a skill that should be practiced in History or English courses, rather than mathematics. The purpose of this paper is to record the results from a research study designed to examine the influence of calculus writing prompts and provide implications for instructors.

Ideas About Writing to Learn Mathematics

In a course designed for mathematics majors, learners explore some mathematics topics using writing prompts. For this study, writing prompts are defined as a situation where a learner explains a mathematical topic so that another learner would understand without simply restating a formula. Many participants in this study expressed disappoint for the idea of using calculus writing prompts to explain their mathematical thinking. This could be due to the lack of experience with writing to explain one's mathematical understanding. Morgan (1998) noted learners in mathematics might complete practice exercises without writing a single sentence to explain their solutions. Learners were asked to explain their mathematical thinking and the thinking of others for various topics in mathematics. Instructors may not perceive any relationship between writing and learning mathematics (Davision and Pearce, 1988; Ntenza, 2006). Learners must be given an opportunity to write about their mathematical thoughts. This can only be accomplished if the instructor is aware of the benefits for having learners articulate their thoughts in writing. Davision and Pearce (1980), further explain when instructor's use structured writing activities regularly the performance of learners improves drastically.

The Purpose of the Study

A study analyzed the mathematical thinking of twenty-three students' responses to various open writing prompts in order to decipher the students' mathematical understanding (Aspinwall and Aspinwall, 2003). The mathematical thinking of the students was classified into four categories: 1) algorithms and computation, 2) limited understanding, 3) utilitarian value, and 4) conceptual understanding. Do learners view mathematics as a procedural process rather than a conceptual process? Or do the views illustrate a limited understanding and/or a utilitarian value? The purpose of the current study was to determine the mathematics majors (n=9) written explanations for the derivative and integral calculus concepts based on the four categories, see table 1.

Table 1 Calculus writing prompts

Prompt 1 (derivative)	A learner in class is confused with the idea of *the derivative*. How would you explain this concept so that the learner gains a greater understanding without simply restating the formula?
Prompt 2 (integral)	A learner claims that the *integral* is used to find the area under the curve by using rectangles but is experiencing difficulty when finding the integrals for her homework assignment. She then says "I don't understand; I know what I'm supposed to find, I just can't find it." Is this learner correct about the concept? How would you help this learner achieve understanding of the concept? Explain.

Discussion

The calculus writing prompts were presented in a courses designed for mathematics majors in the beginning of a spring semester. Various calculus writing prompts were required; however, the derivative and integral writing prompts were of importance for this study. The derivative and integral calculus writing prompt is the first assignment where my learners are asked to think about writing to explain their understanding. The calculus writing prompts for the derivative and integral were given as a homework assignment, where each learner was able to use resources in order to create a well written response in their explanation for the derivative and integral. I have identified responses from three learners who are identified as: Mary, Billy, and Heather (pseudonyms). These learners were selected since their responses represent diverse mathematical understanding. The following sections describe four categorizes of mathematical understanding and the mathematics majors' responses are organized accordingly.

Algorithms and Computation

Algorithms and computation is when a learners' response is illustrated by evidence of a procedure in nature in order to explain the learners' mathematical understanding. For instance, Mary wrote, "say we're given the $f(x) = x^4 - 3x^3$ the derivative of this function is $f(x) = 4x^3 - 9x^2$." She then proceeds to illustrate a procedure in order to find the derivative. Below is Billy's example who describes given the learner another problem to solve, while he watches to determine if the learner has errors in their work.

> First, I would have the learner state the definition of the limit for me $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$. Once that is stated, I would have the learner walk through a problem with me watching, to see if I can see what they aren't understanding. If there is a problem somewhere I would correct it.

Utilitarian Value

Utilitarian value is a response that reflects the usefulness of mathematics, either now or in the future; however, lack a valid response that would lead to a greater mathematical understanding. Evidence of this is demonstrated in Billy's response:

> This learner does have a limited understanding of the integral function, because there are many uses for the integral function not just to find the area under a curve. It is also used to find the volume of a solid rotated about a given point, as well as for optimizing different kinds of formulas. In the real world, there is a science called physics that uses the integral function to determine how much work it takes to do something. Also, farmer brown uses the integral function to maximize the amount of area he can enclose with a certain amount of fencing.

Billy has clearly identified several instances where the integral is useful. He has yet to offer a well written interpretation so that a learner would have a greater understanding for the integral.

Limited Understanding

Limited understanding is a response that lacks conceptual understanding, while identifying more than one procedure. Heather demonstrated a lack of understanding in her response. For instance, Heather's response was "I would explain to the learner that a derivative of *f(x)* is the same as the instantaneous rate of change, or slope, of a function at any

value x." Heather's response for the derivative indicates she understands the concept, however, lack to interpret a well written explanation to help another learner make sense of the derivative. Another example, was Heather's response to the integral writing prompt was:

> This learner is on the right track, but she does not realize that there is more than one way to find the integral of a problem. Using rectangles to find the area under a curve is a good way to work the problem. First, the learner should set a subinterval for the rectangles with in the given interval of the integral. The learner should know how to find the area of a rectangle. Using this, the learner can add the area of the subintervals that he or she created and multiply the sum of the subintervals to the difference of the main interval divided by the number of subintervals. These are all things the learner should know by the time they reach calculus. The learner just needed something that he or she already knows to relate what they are learning.

Heather's response, indicates a limited understanding since using rectangles will not yield the most accurate area under a curve.

Conceptual Understanding

Conceptual understanding is when a learner is able to articulate a well written mathematical explanation. Mary's response indicates conceptual understanding of an integral:

> This learner has the right concept; the integral is the area under the curve of a function. However, her understanding is limited. I would explain to this learner that even though an integral IS the area under the curve, you cannot just draw rectangles and add up the areas because the number of rectangles you draw will affect the answer. Depending on the shape of the curve, you may end up with too much area or too little. But we want to be precise. So in order to find the exact area under that curve we need an infinite number of rectangles.

In Mary's response, she explains to the learner the idea for using rectangles will yield a solution, however, the solution will not be precise. She concludes by helping the learner to understand that an infinite number of rectangles must be used to arrive to an accurate solution.

Responses Evaluated

All learners were able to calculate derivatives and integrals, as demonstrated on various assessments, however, this research project analyzed the learners' ability to provide well written explanations for the derivative and integral calculus concepts. The learners were all able to provide valid elucidations for the derivative and integral concepts, however, not all explanations lead to conceptual understanding. For instance, Heather's response for the derivative and integral showed that she was able to provide a definition for both, however, lacked a concise explanation to help a learner without restating a procedure. Billy and Mary's response for the derivative writing prompt was categorized as procedural understanding, while Mary's response for the integral writing prompt was conceptual understanding and Billy's was utilitarian value, see table 2.

Table 2. Writing prompt (1 & 2) responses for Mary, Billy, and Heather

CALCULUS WRITING PROMPTS	Mary	Billy	Heather
Prompt 1 (derivative)	AC	AC	LU
Prompt 2 (integral)	CU	UV	LU
Key: CU: Conceptual Understanding LU: Limited Understanding UV: Utilitarian Value AC: Algorithms and Computation			

As for all nine learners, their responses varied for both the derivative and integral writing prompts, see table 3. For both the derivative and integral writing prompts, approximately 55.5% of the learner written responses were conceptual understanding & algorithms and computation, while approximately 66.6% were utilitarian value. From the data, it is apparent that the learners view the derivative and integral as procedural mathematics and they are able to articulate the usefulness.

Table 3. Approximate percentages for all nine learner's responses

CALCULUS WRITING PROMPTS	Conceptual Understanding	Utilitarian Value	Limited Understanding	Algorithms and Computation
Prompt 1 (derivative)	3 (33.3%)	2 (22.2%)	1 (11.1%)	3 (33.3%)
Prompt 2 (integral)	2 (22.2%)	4 (44.4%)	1 (11.1%)	2 (22.2%)

Benefits for Learner

Some learners in the study were unable to articulate a well written explanation for the derivative and integral calculus concepts. With further opportunities to express their mathematical thoughts, learners will be able to formulate well written ideas. There are many benefits for allowing learners opportunities to write about their mathematical thinking:

- allow learners to dialog with teacher (Miller, 1992)
- learners express and reflect their attitudes, knowledge, processes, and belief about mathematics (Miller, 1991)
- improve learning (Miller, 1991)
- improve problem solving (Johnson, 1983)
- learners do considerable thinking and organizing of their thoughts (Johnson, 1983; McCarthy, 2008)
- experience expressing mathematical thoughts (Johnson, 1983)
- stimulate thinking about mathematics (Johnson, 1983)
- improve learner writing (Sjoberg, Slavit, Coon, 2004)
- improve ability to make connections to real-life applications with confidence (Sjoberg, Slavit, Coon, 2004)
- allow learners' opportunities to think about mathematical ideas (Porter and Masingila, 2000)
- increased learners' ability to understand higher levels of mathematics (Sjoberg, Slavit, Coon, 2004)

Learners need opportunities to reflect and gather their thoughts about new mathematics learned; and possibly make connections to mathematics already learned.

Benefits for Instructor

Allowing learner's, the opportunity to write their mathematical thinking provides the instructor with valuable information. A list of benefits for the instructor are:

- improve instruction (Miller, 1992)
- allow learners to dialog with instructor (Miller, 1992)
- informal assessment (Miller, 1991)
- writing is for all levels of mathematics (i.e., abstract algebra, and analysis) (Johnson, 1983)

Instructors also acquire information about learner's progress and/or development through allowing learners opportunities to write in mathematics. Furthermore, the instructor can determine struggling learners and/or those who have misconceptions. Some instructors even use writing in mathematics to detect learner's beliefs, attitudes, and/or ability to question the world using mathematics.

Conclusion

In this study, writing prompts were implemented with undergraduate mathematics majors, however, the results of the study yield a greater perspective for higher education instructors. Mathematics majors also need opportunities to explain their mathematical understanding, regardless of mathematics or statistics course. Writing prompts allow learners' the opportunity to write about their mathematical thinking, which will inform the instructor of their learners' mathematical development.

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