ON THE HISTORY OF SOME LINEAR ALGEBRA CONCEPTS: FROM BABYLON TO PRE-TECHNOLOGY

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Abstract: Linear algebra is a basic abstract mathematical course taught at universities with calculus. It first emerged from the study of determinants in 1693. As a textbook, linear algebra was first used in graduate level curricula in 1940's at American universities. In the 2000s, science departments of universities all over the world give this lecture in their undergraduate programs.

The study of systems of linear equations first introduced by the Babylonians around at 1800 BC. For solving of linear equations systems, Cardan constructed a simple rule for two linear equations with two unknowns around at 1550 AD. Lagrange used matrices in his work on the optimization problems of real functions around at 1750 AD. Also, determinant concept was used by Gauss at that times.

Between 1800 and 1900, there was a rapid and important developments in the context of linear algebra. Some theorems for determinant, the concept of eigenvalues, diagonalisation of a matrix and similar matrix concept were added in linear algebra by Couchy. Vector concept, one of the main part of liner algebra, first used by Grassmann as vector product and vector operations. The term 'matrix' as a rectangular forms of scalars was first introduced by J. Sylvester. The configuration of linear transformations and its connection with the matrix addition, multiplication and scalar multiplication were studied first by A. Cayley.

After the 1900s, the improvement of linear algebra was more technique and in a way that more correlative with other disciplines of mathematics than past. Heisenburg used matrix algebra for quantum mechanics. Von Neuman and Turing introduced stored-program computers.

Keywords: linear algebra, history, matrix, vector, determinant, linear equation

Introduction

It is a general and an open-ended question that when and where did mathematics begin? The answer is depended on what do you mean by the word 'mathematics'? Then, we restrict 'mathematics' to logical-deductive tradition. In ancient times, written materials of new mathematical developments have come to known only in a few places (Avital, 1995; Boyer, 1985; Freudenthal, 1991; Jons, 1995 & Katz, 1995). As a scientific discipline, mathematical studies began in the middle of sixth century (Freudenthal, 1991 & Sfard, 1991). The old mathematical materials were registered in mathematical papyrus between 2500-1700 BC. The context in these material were designed with geometry as Pythagorean Theorem and some relations about lines. In the modern ages, mathematical developments had a periodic time points as the golden age of Islamic mathematics in the 11^{th} century and renaissance mathematics in the 16th century.

According many mathematicians, we couldn't see a consensus on the history of linear algebra (Dorier, 2000; Freudenthal, 1991; Katz, 1995 & Kleiner, 2007). Some of them say that linear algebra history starts with matrix and the others say that main starting point is vector spaces. But we see a main linear algebra concept in these approaches that the formulation of linear system of equations and their solutions. In this study we give some basic points of the history of linear algebra concepts. The concepts are linear equations, matrices, determinants, vector spaces.

Linear Equations

System of linear equations are not only the basic concept of linear algebra but also it has a wide using area in many sub branch of mathematics. Moreover, solving of linear equations with two or three unknown and 2-5 equations is a basic part of middle school algebra and college mathematics.

The study of system of linear equations first introduced by the Babylonians around at 1800 BC. For solving of system of linear equations, Cardan constructed a simple rule for two linear equations with two unknowns around at 1550 AD. Cardano (or Cardan), an interesting and colorful personality, published the mathematics book “Ars Magna”. In this book, Cardano with Tartaglia first used the numbers which are known as complex numbers. Lagrange used matrices in his work on the optimization problems of real functions around at 1750 AD. Metric systems, some of mathematicians consider Lagrange as the founder of the metric systems, was used widely in the context system of linear equations. Also, determinant concept was used by Gauss at that times. He formulated the set of errors to be minimized as a system of linear equations.

In ancient China at BC, it was used different ways of solving system of linear equations system (Shen, Crossley & Lun, 1999). These solution methods were not fully different each other and some of them could be used only for two or three equations whit the same number unknowns. Also ancient Chinese made some correlations between the number equations and the number of unknowns for unique solutions and more than one solution. According to the authors, there are many examples, application problems and their solutions in the old text books.

Matrices

The history of matrix goes back to ancient times at BC. But we can say that the word “matrix” was not used clearly until 1855. İt was born from the formulating of the system of linear equations by English mathematician Sylvester. Sylvester and Cayley published their famous text on matrices at these times. In this text, they used matrices by means of system of linear equations (as seen below) and introduced some theories and relation in the context of matrix as matrix addition, multiplication and inverse.

$$X = ax + by + cz$$

$$Y = dx + ey + fz$$

$$Z = kx + ly + mz$$

and

$$(X, Y, Z) = \begin{pmatrix} a & b & c \\ d & e & f \\ k & l & m \end{pmatrix} (x, y, z).$$

Gauss gave some elementary formulations for solving system of linear equations but do it without using matrices. When the mathematicians focused on studying determinants, this motivation turned out to be the term ‘matrix’. At the end of the 1900’s, Matrices were used not simply for solving systems of linear equations, but also in concepts of liner algebra. Moreover, they have wide speared of using in physics, engineering and some social area as phycology. By the effective using of technology in linear algebra, all solution methods created by Cayley, Gauss, Sylvester moved backward benefiting them in linear algebra lectures.

Determinants

J.L. Dorier (2000) stayed that the concept ‘determinant’ was used separately in different two places in both Asia and Europe. Also Dorier stressed that in ancient times at BC for solving system of linear equations was benefited from some geometric figures as called counting boards to represent problems. Mikami (1974) claimed that a special version of determinant to solve nonlinear system of equations at 1700’s. According to him, this solution method was formulated in a way that registered in a rectangular array the positions of coefficients of equations. Mikami redesigned this formulation from the historical text for 2x2 and 3x3 square matrices (figure1)

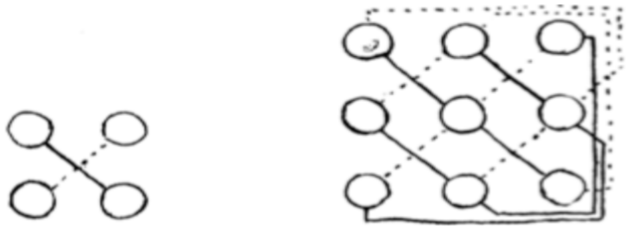

Figure 1. Mikami's redesign of determinant of matrices 2x2 and 3x3.

Determinants were used by Leibnitz, known as the founder of calculus, in 1700's. Lagrange used determinants in his working on Lagrange multipliers about maximum and minimum problems. In 1750's, Cramer formulated solution of some special system of linear equations, called 'Cramer's rule', and used determinant as a variable in this characterization.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

By analyzing historical text, we can say that there are four mathematical subject which are directly connected determinants; matrices, solution of system of linear equations, elimination of a matrix of a system of linear equations and linear transformations. But, it is not clear to give any definite connection between general determinant formulas and the other linear algebra subjects. After the Cramer, determinants of the square matrix were used first time as known modern characterization.

Vector Spaces

In geometrical format, vector spaces in R^2 and R^3 were firs represented by Descartes and Fermat. Italian mathematician Peano defined the known modern definition of vector space concept in 1900's (Katz, 1995). After Peana, another Italian mathematician Weyl used vector spaces more efficiently and attractively in his studies that registered 'vector spaces and transformations view' of linear algebra and 'our axioms characterize the basis of our operations in the theory of system of linear equations by composing the coefficients of the unknowns in a system of linear equations as vectors. Lebesgue used real function spaces as vector space in his work. After than Lebesgue, Hilbert and Banach formulated and developed functions spaces as 'Hilbert and Banach spaces'.

Bourbaki (1975) stayed that Hawkin suggested a reasonable and methodological presentation for the matrix theory, from starting with systems of differential equations by lagrange to theoretical presentation by Weierstrass. According to Baurbaki, the theory of vector spaces had the same methodological procedure. For the explicit formulation of vector spaces, studies on the systems of linear differential equations played basic role (Helgason, 977). As mentioned above, Piano made the axiomatic formal definition of vector spaces over real number fields. We understand that the study of differential equations helped to formulating of vector space concept. Bourbaki (1975) stated that the study of function spaces formulated by Hilbert, but he focused on the kernels of integral equations and generalizations of their matrices. Also, we can say that the difficulty and complexity of infinite vector space directed mathematicians to focusing on the finite dimensional vector spaces.

Famous Frenchman mathematician Galova used the finite dimensional vector spaces to formulate his famous theory called Galois Theory. The linear combinations of algebraic numbers was an extension of the rational numbers. This mean that the defining of vector space over the field of real numbers was a methodological result (Kiernan, 1971). Kiernan expressed that Dedekind motivated seriously at the structure of fields, the number theory. According to him, by the effect of this motivation, he formulated the similarity of algebraic numbers and algebraic function theory. He remarked that the vector space theory played an essential role for the modern reformulation of Galois Theory.

Result

It is not possible to give all historical developments with a research paper about linear algebra concepts. We only dealt with some basic point of linear algebra concepts in this paper. More general search could be available from history of linear algebra text books or research papers (Benzi, 2009, Dorier, 2000 & Kleiner, 2007). It is clear that there is a powerful connection between mathematics concepts especially linear algebra concepts. And we can see this connection in the ancient times in which mathematicians formulated or characterized this concepts. Maybe, mathematicians had become infatuated with the connection of the concepts of mathematics.

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