On Scalar Quark Leakage through the Power-Law and Logarithmic Confining Potentials in the Klein-Gordon Equation

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Abstract: Motivated from a phenomenological viewpoint, three confining potentials have been studied in the Klein-Gordon equation framework. In particular we study the phenomena of quark leakage for these potentials. The transmission coefficient values have been obtained for all the potentials, using WKB [Wentzel, Kramers & Brillouin] method and compared with potentials already discussed by previous authors. We observe that one of the potentials considered by us strongly supports the existence of free quarks because of the comparatively large values of transmission coefficients, as predicted by our model.

Keywords: Confining potentials; tunnelling phenomenon; quark leakage; transmission coefficients.

Introduction

Mankind has sought the elementary building blocks of matter ever since the days of the Greek philosophers. Over time, the quest has been successively refined from the original notion of indivisible “atoms” as the fundamental elements to the present idea that objects like quarks lie at the heart of all matter.

With the new discoveries being made at the Large Hadron Collider at CERN, interest in hadron spectroscopy has still not waned. Many attempts have been made in the past as well in the present to study the hadron spectroscopy, using both the non-relativistic Schrödinger equation and the relativistic wave equations viz., the Klein-Gordon and the Dirac equations.

It may be noted that the model involving fractionally charged quarks was proposed by Gell-Mann (1964) and Zweig (1964) to account for the explosion of subatomic particles discovered in accelerator and cosmic ray experiments during 1950s and early 1960s. Their model won acceptance because of a few semiquantative tests e.g., a large weight of circumstantial evidence and many quantitative facts about strong interactions which it apparently explains. Since then, however, there have been many unsuccessful attempts to find quarks at accelerators, in sea water, in rocks and in cosmic rays. Though La Rue et al (1977) have claimed that they have found some evidence for quarks in their super-conducting levitation experiment involving niobium pellets.

There have been many attempts to understand the physical mechanism of quark confinement. However, none of them is completely convincing and satisfactory. Even the most ambitious attempts based on quantum chromodynamics (QCD) provide rather vague explanations of the mechanism of confinement. The problem was earlier attacked also with the help of less ambitious but more plausible models like naïve strings, bags, suitable potentials *etc*. The problem of confinement was also tried, treating hadrons as systems of quark solitons with some suitable non-linear interactions (Werle, 1993).

Kang and Schnitzer (1975) have calculated meson spectra, using a potential function $ar+b$ as the fourth component of a four-vector in the Klein-Gordon equation. The quark-antiquark bound-state energy values (which correspond to meson masses) were calculated using WKB approximation. Gunion and Li (1975) have studied the same potential as a Lorentz scalar in the Klein-Gordon and Dirac equations. The motivation of their using the linear potential came from field-theoretic arguments. Sharma with his collaborators (2008, 2007, 2004, 2003, 1998, 1988, 1984, 1983, 1982, 1982, 1980) has extensively studied quark confinement and have calculated bound-state spectra for both the light and heavy mesons e.g., the bound-states for $c\bar{c}$, $b\bar{b}$, $s\bar{s}$ etc., spectra have been calculated by them. Recently Sharma et al (2000, 2003) have also evaluated the spectra of $t\bar{t}$, the so called toponium meson which has yet to be observed. The heavy top quark was detected by two teams working at Fermi National Laboratory, the first one the so called CDF team (Abe et al. 1995) reported its mass as $176\pm13 GeV$ and the other team the Dø collaboration (Abachi et al. 1995) estimated its mass as $199\pm30 GeV$. The top quark appears to be a point-like particle: it has no internal structure that one can discern.

In the above context it is always interesting to study the phenomenon of quark leakage, using different potential models. The application of linear (Kang & Schintzer, 1975) and the oscillator potentials (Ram & Halasa, 1979) in a relativistic framework such as the Klein-Gordon equation gives rise to the phenomenon of tunnelling and hence to the leakage of quarks. Ram (1978) has discussed numerically the tunnelling phenomenon for a linear potential and also Kajwadkar & Sharma (1983) have obtained transmission coefficients for two potentials viz., logarithmic and cubic power potentials.

To get a deeper insight into the phenomena of quark leakage, we deal in this paper numerically, with the quark leakage for three different potential models. It may be interesting to note that all the three potentials have already been successfully applied in explaining the meson spectroscopy.

Potentials considered by us here are:

(I)– A fractional power potential (Martin, 1980) given by

$$V_1(r)=g_1 r^{0.1}-V_{0,} \tag{1}$$

(II)- A power law potential (Sharma &Sharma, 1984) of the form

$$V_{II}(r)=g_1(r)^{\frac{m_0}{2m_q}}-V_0, \tag{2}$$

with $m_0=1GeV$ and m_q being the quark mass.

(III)- A logarithmic potential of the type (Jena, 1983)

$$V_{III}(r)=g_1\log(1+r)-V_0 \tag{3}$$

Here in all the three potentials (I), (II) and (III), $g_{1,} V_0 > 0$.

In the following section 2, we study the tunnelling phenomenon for all the three above mentioned potentials by evaluating expressions for their corresponding transmission coefficients.

Finally, in section 3, we give a brief discussion on the results obtained by us in section 2.

Theory

The motion of a quark in one-body central potential $V(r)$ is governed by the following relativistic Klein-Gordon equation $(c=\hbar=1)$:

$$\left(-\nabla^2+m^2\right)\psi(r)=\left[E-V(r)\right]^2\psi(r). \tag{4}$$

The radial part of this equation can be simplified to the form

$$\frac{d^2U(r)}{dr^2}+2m\left[\overline{E}-V^{eff}(r)\right]U(r)=0. \tag{5}$$

Where

$$U(r)=rR(r),$$

$$\psi(r)=R(r)Y_l^m(\theta,\phi)$$

$$\overline{E} = \frac{E^2 - m^2}{2m}, \qquad (6) \qquad \text{and}$$

$$V_{eff} = \frac{1}{2m}\left[\frac{l(l+1)}{r^2} + 2EV - V^2\right]$$

Following Merzbacher (1970), we use the WKB approximation and obtain the following expression for the transmission coefficients of s-states $(l = 0)$

$$T = \frac{4}{\left(2\theta + \frac{1}{2\theta}\right)^2}. \qquad (7)$$

Here

$$\theta = \exp\left[\int_{r_1}^{r_2} k(r)\,dr\right] \qquad (8)$$

With

$$k(r) = \left[2m\left(V^{eff} - \overline{E}\right)\right]^{\frac{1}{2}}, \qquad (9)$$

where r_1 and r_2 are the roots of the equation $k(r) = 0$.

Results and Discussion

The values of these transmission coefficients obtained for different values of m and g_1 are depicted in Tables 1 and 2. For the purpose of comparison, the transmission coefficients for linear (Kang & Schnitzer, 1975); cubic and logarithmic potentials [of the type $g \ln \frac{r}{r_0}$] (Kajwadkar & Sharma, 1983) have also been shown in Table 1. It may be noted that our equations (4), (5) and (6) can also be used in the calculations of transmission coefficients for a system consisting of quark and anti-quark but with the following substitutions (Kang &Schnitzer, 1975; Ram & Halasa, 1979; Iyer &Sharma, 1982; Sharma & Iyer, 1982):

$$V^{eff} = \frac{1}{2m}\left[\frac{l(l+1)}{r^2} + \frac{1}{2}EV - \frac{1}{4}V^2\right] \qquad (10)$$

and

$$\overline{E} = \frac{\frac{1}{4}E^2 - m^2}{2m} \qquad (11)$$

The values of these transmission coefficients for different mesons along with parameters actually used in obtaining the meson spectra, are shown in Table 3.

Principal observations made in this paper are:

(A) The smaller the effective power of the potential, the lower is the value of the transmission coefficient. Consequently the probability of leakage of quark is higher for the potential with larger effective power (Table 1).

(B) For potential (3), the transmission coefficients are comparable with those obtained for the logarithmic potential (Kajwadkar & Sharma, 1983), see- Table 1.

(C) The fall in the value of transmission coefficient with increasing mass is steeper for smaller values of effective power (Fig. 1). From Fig. 1 and Table 3, it is also evident that the transmission coefficients for potential (1) are very small while for the potential (2) they are much higher.

(D) The effect of the variation in g_1 is perceptibly larger for potential (1). For this potential, there is a sharp rise in the value of transmission coefficient with increasing value of g_1 (Fig. 2). For large values of g_1, however, the transmission coefficient for potential (1) attains almost a constant value. While for potential (2), it is nearly constant for all values of g_1. For potential (3), the variation in the values of the transmission coefficients with the values of g_1 lies in between those obtained for potentials (1) and (2) within the range considered.

(E) Potential (2) supports strongly the existence of free quarks because of comparatively large values of transmission coefficients predicted by this model. Potential (1) on the other hand, gives rise to a very small possibility of quark leakage. Particularly, as can be seen from Table 3, leakage of quarks for all the three mesons are practically zero for potential (1). For ρ meson, values of transmission coefficients predicted for potential (2) are higher than those obtained for linear and oscillator potentials. For $\phi(s\bar{s})$ mesons, transmission coefficients predicted by potential (2) are independent of energy [since m_s has been chosen to be equal to 0.5 GeV, consequently potential (2) becomes a linear potential]. Similar observations were found by Kang and Schnitzer (1975) and Ram and Halasa (1979) for the linear potential.

(F) It may be of interest to note that the potential (2) for meson state ρ with the parameters chosen gets transformed to an oscillator potential. The calculated values of T are in agreement with the corresponding values for the oscillator potential calculated by Ram & Halasa (1979) [see Table 3]. Similarly, for ϕ meson states, the parameters chosen transform potential (2) to a linear potential and the values of calculated T drop and agree with the values calculated by Kang &Schnitzer (1975).

(G) From Fig. 1, we see that as the mesons get heavier, it becomes harder for them to leak through the confining potential barrier. From Fig. 2 we observe that at low g_1 values, the transmission coefficient associated with potential (1) decreases more rapidly as the meson mass increases, followed by those associated with the third and lastly the second potentials. However, at higher g_1 values, the transmission coefficient that decreases the greatest is for the third potential, with the first being the least decreasing with increasing meson mass.

(H) From Fig. 2, we observe that the value of transmission coefficient increases as g_1 increases. For potential (2), the transmission coefficient rises from low values of g_1, approaching some asymptotic value as g_1 increases. For the first and the third potentials, the transmission coefficient remains very close to zero as g_1 increases, after which it jumps to the common asymptotic value. This “jump” occurs at greater values for the first potential than for the third. For heavier mesons, the transmission coefficient increases much more slowly to attain the common asymptotic value.

Table 1: Transmission coefficients T for the three potentials (I), (III) and (III).

For potential (I) $g_1 = 1.0(GeV)^{1.1}, V_0 = 0$ and $E = 1.0\,GeV$

For potential (II) $g_1 = 1.0(GeV)^{\frac{(1+2m)}{2m}}, V_0 = 0,$ and $E = 1.0\,GeV$

For potential (III) $g_1 = 1.0\,GeV, V_0 = 0,$ and $E = 1.0\,GeV$

Quark mass m (GeV)	T for potential (I) $V_I = r^{0.1}$	T for potential (II) $V_{II} = r^{m_0/2m}$ $m_0 = 1GeV$	T for potential (III) $V_{III} = \log(1+r)$	T for Linear potential (Ram, 1978) $\alpha = 1$	T for cubic potential (Kajwadkar, et al, 1983)	T for logarithmic potential (Kajwadkar *et al*, 1983)
0.1	0.520	0.638	0.609	0.63	0.63	0.61
0.2	0.190	0.622	0.518		0.62	0.53
0.3	8.555×10^{-3}	0.578	0.382	0.53	0.60	0.40
0.4	4.622×10^{-6}	0.498	0.235	--	--	--
0.5	3.292×10^{-13}	0.380	0.118	0.37	0.55	0.14
0.75	3.042×10^{-79}	0.0834	8.008×10^{-3}	--	--	--
1	≈ 0	3.346×10^{-3}	1.285×10^{-4}	0.04	0.27	$4.6\text{x}10^{-4}$
1.25	≈ 0	1.047×10^{-6}	3.731×10^{-7}	--	--	
1.5	≈ 0	5.647×10^{-15}	1.249×10^{-10}	--	--	
2	≈ 0	1.027×10^{-85}	1.313×10^{-21}	--	--	

Table 2: Transmission coefficients T for potentials (I), (II) and (III) for different values g_1. Other parameters are:
$E = 1.0\,GeV,\ m = 1.0\,GeV,$ and $V_0 = 0.$ Units of g_1 for different potentials are same as used in Table I.

Quark mass (GeV)	$g_1 = 0.9$			$g_1 = 2.0$			$g_1 = 3.0$		
	T for potential $V_I = r^{0.1}$	T for potential $V_{II} = r^{m_0/2m}$	T for potential $V_{III} = \log(1+r)$	T for potential $V_I = r^{0.1}$	T for potential $V_{II} = r^{m_0/2m}$	T for potential $V_{III} = \log(1+r)$	T for potential $V_I = r^{0.1}$	T for potential $V_{II} = r^{m_0/2m}$	T for potential $V_{III} = \log(1+r)$
0.1	0.3307	0.6377	0.6016	0.6399	0.6380	0.6306	0.6400	0.6382	0.6347
0.2	0.01146	0.6209	0.4903	0.6394	0.6261	0.6025	0.6400	0.6282	0.6188
0.3	1.1883×10^{-6}	0.5745	0.3310	0.6382	0.5992	0.5562	0.6400	0.6080	0.5924
0.4	4.9984×10^{-16}	0.4860	0.1763	0.6354	0.5567	0.4937	0.6399	0.5795	0.5560
0.5	1.5854×10^{-36}	0.3555	0.0720	0.6292	0.5016	0.4188	0.6398	0.5464	0.5104
0.75	≈ 0	0.0557	2.310×10^{-3}	0.5729	0.3452	0.2187	0.6388	0.4675	0.3678
1	≈ 0	8.8124×10^{-4}	1.134×10^{-5}	0.3529	0.2141	0.07743	0.6344	0.4138	0.2186
1.25	≈ 0	1.6490×10^{-8}	5.3099×10^{-9}	0.0227	0.0840	0.01828	0.6149	0.3396	0.1046
1.5	≈ 0	2.5034×10^{-20}	1.124×10^{-13}	2.399×10^{-6}	0.0168	2.8730×10^{-3}	0.5551	0.2587	0.04003
2	≈ 0	≈ 0	4.142×10^{-29}	8.9564×10^{-46}	4.8777×10^{-6}	1.9045×10^{-5}	0.1526	0.0854	3.0234×10^{-3}

Table 3: Transmission coefficients T for different Mesons. Parameters used are:
For potential $V_I(r)$:
$5.996(GeV)^{1.1}, V_0 = 7.01GeV, m_u = 0.39GeV, m_s = 0.52GeV$ and $m_c = 1.806GeV$.
For potential $V_{II}(r)$:

(i) **For ψ mesons-**

$m = 2.0GeV, m_0 = 1.0GeV, g_1 = 2.365(GeV)^{1.25}$ and $V_0 = 3.8833GeV$.

(ii) **For ϕ mesons-**

$m = 0.5GeV, m_0 = 1GeV, g_1 = 0.2725(GeV)^2$ and $V_0 = 1.089GeV$.

(iii) For $\rho-$ mesons-

$m = 0.25GeV, m_0 = 1.0GeV, g_1 = 0.037(GeV)^3$ and $V_0 = 0.761GeV$.

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$V_I = r^{0.1} - V_0$		$V_{II} = r^{m_0/2m} - V_0$		Linear potential (Kang & Schnitzer, 1975)		Oscillator potential (Ram & Halasa, 1979)	
Meson State (Energy GeV)	T	Meson State (Energy GeV)	T	Meson state (Energy GeV)	T	Meson state (Energy GeV)	T
ρ(1s) (0.77) ρ(2s) (1.60)	2.12×10^{-6} 1.92×10^{-15}	ρ(1s) (0.767) ρ(2s) (1.60) ρ(3s) (2.2825)	0.4175 0.4580 0.4789	ρ(1s) (0.767) ρ (2s) (1.60) ρ(3s) (2.228) ρ(4s) (2.754)	0.22 0.22 0.22 0.22	ρ(1s) (0.776) ρ(4s) (2.911)	0.40 0.49
Φ (1s) (1.80)	2.65×10^{-34}	Φ (1s) (0.767) Φ (2s) (1.60) Φ (3s) (2.2825)	0.134 0.134 0.134	Φ (1s) (1.019) Φ (2s) (1.806) Φ (3s) (2.410) Φ (4s) (2.92)	9×10^{-13} 9×10^{-13} 9×10^{-13} 9×10^{-13}	Φ(1s) (1.022) Φ(4s) (3.065)	0.12 0.24
$\psi(1s)$ (3.095)	0	ψ (1s) (0.767) ψ (2s) (1.60) ψ (3s) (2.2825)	0	ψ (1s) (0.767) ψ (2s) (1.60) ψ (3s) (2.2825) ψ (4s) (2.754)	4×10^{-37} 4×10^{-37} 4×10^{-37} 4×10^{-37}	Ψ (1s) (3.179) Ψ(4s) (4.656)	4×10^{-1} 10^{-9}

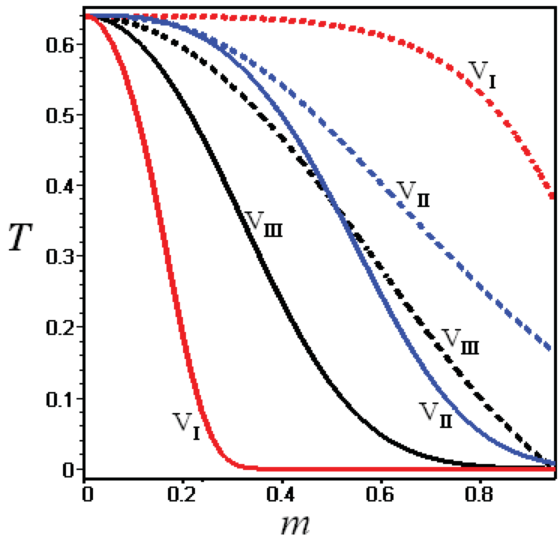

Figure 1: The variation of transmission coefficient T with the quark mass. The solid plots correspond to $g_1 = 1.0GeV$ while the dashed curves are for $g_1 = 2.0GeV$.

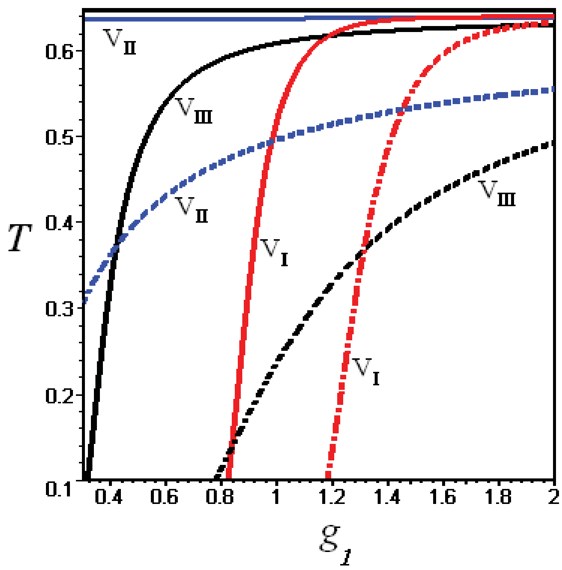

Figure 2: The variation of transmission coefficient T with g_1 . The solid plots correspond to $m = 0.1GeV$ while the dashed curves are for $m = 0.4GeV$.

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