A Case Study on Teacher Instructional Practices in Mathematical Modeling

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ABSTRACT

There is in fact a tendency in several countries to include more mathematical modeling in curriculums. Mathematical curriculum in Turkey focuses on the principle of "every child can learn mathematics". From this perspective; more importance is given to modeling from the sixth grade to eighth grade mathematical curriculum. Accordingly, prospective mathematics teachers are required to be trained for preparing teaching environments appropriate for mathematical modeling. In this context, 33 prospective mathematics teachers are trained of mathematical modeling based teaching. 17 of them are selected randomly for modeling based teaching applications. Modeling based teaching carried out with modeling tasks developed by 17 prospective mathematics teachers at the end of training. This study is a case study of one of these 17 cases is selected for this study to provide descriptive information about instructional practices in mathematical modeling. Modeling task is applied on randomly selected 38 8th grade students in a practicing school. Both quantitative and qualitative data collection tools are used. The study presents information about instructional practices with data drawn from classroom observations and scoring rubrics.

Keywords: *Mathematical modeling; prospective mathematics teacher; teacher instructional practices; case study.*

INTRODUCTION

Mathematics is a systematic way of thinking which enhance solution to real world statements and problems with mathematical modeling. Blum & Ferri (2009), defines "modeling competency" as the ability to construct models by appropriately carrying out definite steps as well as analyzing or comparing given models. Modeling can be determined as transformation of a problem into mathematical notions and representations (Burkhardt & Pollak, 2006; Niss, 1987; Kaiser; Blomhǿj & Sriraman, 2006). Mathematical modeling is meant to help students' better understand the world, support mathematics learning (motivation, concept formation, comprehension, retaining), contribute to develop various mathematical competencies and proper attitudes, contribute to create an adequate picture of mathematics, namely using enough mathematics. In this context, purposes of mathematical modeling are; enable students make predictions, explain problems, describe and understand different situations in the real world (Galbraith & Catworthy, 1990).

Mathematical modeling is an important component of professional training, which is similar in all areas, particularly in mathematics education. The incorporation of mathematical modeling in mathematics education provides a learning environment (D'Ambrosio, 2009). There are many characterizations or modeling cycles of modeling process (Burkhardt, 1981; Edwards & Hamson, 1989; Hirstein, 1995; Berry&Houston,1995; Borromeo, Ferri,2006; Galbraith & Stillman, 2006; Pollak, 1979; Verschaffel, Greer & De Corte, 2000). In the year of 1989, Standards of National Council of Teachers of Mathematics depicted modeling as an iterative, five step process: 1.construction of a simplified version of the initial problem situation, 2. construction of a mathematical model of the simplified problem, 3. identifying solutions within the framework of the mathematical model, 4. interpreting these solutions in terms of the simplified problem situation, 5. verifying that the solutions of the idealized problem are the solutions to the initial problem. The NCTM Standards' (1989) characterization of mathematical modeling is given in Figure 1. One of the process models to describe modeling activities is the modeling cycle proposed by Blum& Leiss (2007). In an idealised form, the solution process for a modeling problem can be characterized by a seven-step sequence of activities: (1) understanding the problem and constructing an individual "situation model"; (2) simplifying and structuring the situation model and thus constructing a "real model"; (3) mathematising, i.e. translating the real model into a mathematical model; (4) applying mathematical procedures in order to derive a result; (5) interpreting this mathematical result with regard to reality and thus attaining a real result; (6) validating this result with reference to the original situation; if the result is unsatisfactory, the process may start again with step 2; (7) exposing the whole solution process. From this point of view, the modeling process is made up of seven steps. Distinguishing between these steps is helpful for reconstructing the modeling processes used by students when solving mathematical problems. However, students' actual processes are typically not linear but rather jump back and forth several times between mathematics and reality (Borromeo Ferri, 2007; Leiss, 2007).

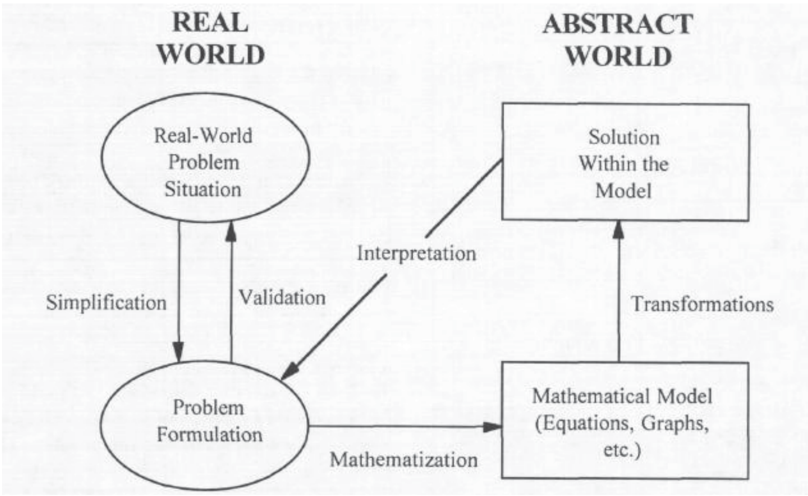


Figure1. The NCTM Standards' (1989) characterization of mathematical modeling

Demanding transfer processes between reality and mathematics are the core of modeling activities (Blum, Galbraith, Henn & Nis (2007); Pollak, 1979). Seven-step model developed by Blum & Leiss (2007) is given in figure 2. There are seven steps passed through in this cycle such that *1.understanding, 2.simplifying/structuring, 3.mathematising, 4. working mathematically, 5.interpreting, 6. validating and 7. exposing* (Blum&Borromeo Ferri,2009). One characteristic advantage of this seven-step modeling cycle is the separation between constructing a situation model, a real model and a mathematical model. This allows for distinguishing between difficulties in understanding the given situation, in simplifying and structuring the information extracted from the situation, and in choosing a suitable mathematical description of the situation during students' solution processes, and thus helps teachers in choosing appropriate, well-aimed and adaptive interventions especially in the critical translation phase at the beginning of the modeling process (Schukajlow et. al, 2011).Generally speaking, the seven-step cycle described below is both sufficiently detailed to capture the essential cognitive activities taking place in actual modeling processes and sufficiently simple to guide the necessary observations and analyses in a parsimonious way (Schukajlow et. al, 2011). A cognitive analysis of modeling process gives a model of the modeling cycle. Modeling cycle can look like algorithmic process, but indeed it is not. Especially the construction process of modeling is challenging as it include formulating a problematic situation. The process requires selection of appropriate variables, determining connections between these variables, developing a mathematical model related to these variables and connections, and testing the model and its applications (Blum & Niss, 1991).

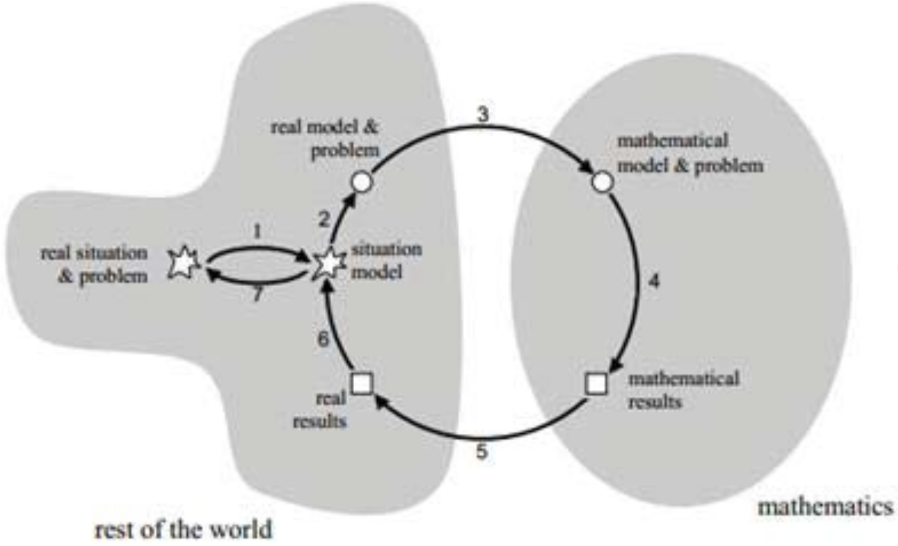


Figure 2. Modeling cycle (Blum&Leiss, 2007)

Underlying all of these justifications of modeling, are the main goals of mathematics teaching in secondary schools. In this context, there is in fact a tendency in several countries to include more mathematical modeling in the curriculum. The basic purpose of involving mathematical modeling in secondary school curriculums is to encourage students make connections between mathematics and the real world. According to the mathematics educators, students have opportunities to use and apply mathematics through mathematical modeling (Blum & Niss, 1991). Mathematics classes that are designed in the form of using mathematical modeling, give students chance to use mathematics actively. Students with mathematical modeling competencies learn and develop mathematical concepts very well which makes important contribution to their mathematical experiences outside school (Aydın, 2008).

Mathematical curriculum focuses on the principle of "*every child can learn mathematics"* in our country. Mathematics curriculum has important attainments on training individuals; students who learn through these curriculums generally have the ability to apply mathematics in their lives, solve problems, share solutions and thoughts, work as a team member, have self-confidence in mathematics and develop positive attitudes towards mathematics (Ministry of National Education, 2009). From this perspective, importance of modeling in 6th, 7th and 8th grade mathematical curriculum is getting increasing. In this case, prospective mathematics teachers who will teach mathematics in future are required to be prepared to understand mathematical teaching environments appropriate for modeling. Modeling has crucial contributions to development of technical processes and technology because of its simple application on all areas of life. If this phenomenon can be learnt as early as secondary school, high school will be able to evaluate everything mathematically in their lives and will be more successful in mathematics classes. According to the results of PISA 2006, students all over the world experience problems with modeling tasks (EARGED, 2010).

In this context, for mathematics education, the importance of prospective mathematics teachers' using the real world problems and carrying out mathematical modeling is increasing. Prospective mathematics teachers are required to be skillful at identifying and developing mathematical modeling. The aim of this study is provide descriptive information about instructional practices in modeling based teaching. Following three research questions guide this inquiry:

1. How are the prospective mathematics teacher's skills in preparing daily lesson plan?

2. How are the prospective mathematics teacher's skills in teaching practice on modeling based teaching?

3. How are the eighth grade students' achievements on modeling based teaching?

METHODOLOGY

The study is a case study over 8^{th} grade students who were involved in a modeling based teaching by the prospective mathematics teacher (PMT). The PMT is selected randomly from prospective mathematics teachers who have 3-month training of mathematical modeling. In this study quantitative and qualitative data are collected and analyzed descriptively.

Participants and Setting

The study is carried out in the academic year of 2010-2011. The research is applied on randomly selected 38 eighth grade students in a practicing school. Modeling based teaching is implemented according to "The Daily Lesson Plan" designed by the PMT. The PMT prepares "The Daily Lesson Plan" for organization of modeling based teaching before the teaching practice. Format of daily lesson plan consists of three parts. In the "*formal partition"*: date of practicing, school of practicing, grade, learning field, sub-learning field, gain, time, learning strategy-method and techniques, materials get involved . In the "*preparatory activity*": there is an activity that measures the readiness and is effective in preparing students for transition to the modeling task. And "*processing partition"* requires identifying a modeling task which is important, appropriate for the students' grade, learning field, sub-learning field, gain and modeling process. In this study the plan is aiming at ensuring students achieve the gain of "*Estimating surface areas of geometric objects by using strategy"* in 8^{th} grade mathematics curriculum. Students are separated into 8 small groups (four or five students per each group) based on friendship and academic results evaluated by the PMT and students' mathematics teacher. Homogeneous distribution between the groups and heterogeneous distribution within the groups are achieved. Modeling based teaching starts with *preparatory activity* in order to measure their previous knowledge about geometric objects; then modeling tasks were given to each group in worksheet format which is easy to work on. The PMT acts as cognitive guide and offered scaffolding when the situation required intervention; for example when students weren't able to understand the problem situation. Groups completed the modeling tasks in given time and presented their solutions on the board. All of the solutions were discussed by the whole class. Session lasted about an hour. Modeling task used in the class is given in Table 1.

Table 1: Haydar Paşa Railway Station Problem

Haydar Paşa Railway Station was established on an area of $2525m^2$, and 6200 m^2 of coating material was used in the construction. Haydar Paşa railway station is the first door (station) of Istanbul which opens to Anatolia and to the Middle East. It has been serving to various people since May 30, 1906.

Unfortunately, some parts of Haydar Paşa Railway Station burnt in the fire on November 28, 2010. If you were the engineer who designs the burnt roof how you would design? Which geometric shape the designed roof would look like? The designed roof needed to be robust, and coated with a special material. The cost per 1 m^2 of this material is 1 Turkish Liras. What would be the cost of the designed roof?

Data Collection Tools

In this study "The Analytic Scoring Rubric for Evaluation of the Daily Lesson Plan (ASRE-DLP)", "Observation Form for the Mathematical Modeling Process (OF-MMP)" and "The Analytic Scoring Rubric for Evaluating Mathematical Modeling Process (ASRE-MMP)" are used to collect data. ASRE-DLP was designed to evaluate the skills of the PMT in preparing daily lesson plan by the researchers. Criteria of the process in daily lesson plan are: *1. determining the preparatory activity, 2. identifying the modeling task, 3. authenticity, 4. visual design, 5. conducting research, 6. checking spelling and grammar and 7. determining the amount of time*. In this instrument, a scoring system including three types of points (1, 2 and 3) is used. In this context, 1 point is given for an inadequate approach and 3 point is given for a truly approach according to the desired situation or to demonstrate an adequate level approach. According to this system, 7 is the lowest score and 21 is the highest score. Success levels are formed in three parts as Bukova Güzel & Uğurel (2010) identify in their study: Preliminary (in this level score is between 7 and 10.4), moderate (in this level score is between 10.5 and 17.4) and high success (in this level score is between 17.5 and 21). A sample criterion of ASRE-DLP is given in order to illustrate scoring system and success levels.

Criterion	Success levels Preliminary level (1)	 Moderate (2)	 High success (3)	Score
Authenticity	Activities are ordinary and similar	Activities are partly original and made by inspiration of similar ones.	Activities are original and different	

Figure 1. A sample criterion of ASRE-DLP

To evaluate the skills in instructional practices of PMT, an observation form named as OF-MMP is used in this study. The behaviors that makes observed environment workable are chosen to prepare an observation form (Yıldırım & Şimşek, 2005). It is important to reveal clearly of behaviors which are needed to observe to what extent. In this context, a check list is prepared. Criteria are taken into consideration to allow monitoring of modeling process as a multi-faceted observation in creating this checklist. Communication, model, mathematical context and evaluation are determined as criteria. This checklist reorganized to allow monitoring the process from the beginning of the course until the termination of the course and became the pre-form. Observation form has been rearranged with regard to the recommendations from the observers, the qualitative data analysis applications and finalized in accordance with expert analysis. Modeling based teaching is also video recorded during the session. Students' model development efforts in the modeling task named as Haydar Paşa Railway Station Problem and the PMT's interventions are monitored by an observer. Therefore the focus is on the PMT's skills in instructional practices based on mathematical modeling.

(ASRE-MMP) is designed as a scoring rubric for analyzing of 8th grade students' achievements on modeling based teaching. This scoring rubric was developed by researchers with the model of Blum & Leiß (2007); criteria of Herget and Torries-Skoumal'ın (2007); six levels of Ludwig & Xu (2008); competences of De Terssac(1996), Maaß(2006) and Berry & Houston(1995); and student activities of Kim & Kim(2010). Criteria of (ASRE-MMP) are determined as *1.understanding, 2.simplifying/structuring, 3.mathematising, 4. working mathematically, 5.interpreting, 6. validating and 7. exposing* and to measure these criteria a scoring system including three types of points (1, 2 and 3) is used. This instrument's scoring system and success levels show similarities with ASRE-DLP.

For the scope validity of the scoring rubrics, experts' opinions who have profession in educational programs-instruction and measurement - evaluation are considered. Expressions in scoring rubrics are designed in line with experts' opinions and aim of the research. Reliability of the scoring rubrics is analyzed as the percentage of coherence of researchers' scorings. Kappa statistic is used to determine the percentage of coherence between two or more evaluators. Kappa coefficient ranges from -1 to +1. If kappa coefficient is zero, there will be random coherence. If kappa coefficient has negative values, this will be worse than random coherence. +1 represents an excellent coherence. If kappa coefficient ranges between .40 and .75, this means a reasonable coherence and greater than .75, this means that there is an excellent coherence (Şencan, 2005, pp.265-267). In ASRE-DLP, percentage of observed coherence is 0.85 and percentage of coherence with chance is 0.38 and in ASRE-MMP, these values are similarly 0.85 and 0.32. In this context, kappa coefficients are calculated and found to be 0.76 for ASRE-DLP and 0.78 for ASRE-MMP. As a result, there is an excellent coherence for both ASRE-DLP and ASRE-MMP. The percentages of coherence in subscales of the ASRE-MMP are calculated and Cohen's Kappa coefficients are respectively as K_1= 0.60, K_2 =0.81 and K_3 = 0.79, K_4= 0.72, K_5=0.79, K_6= 0.79 and K_7=0.79. Therefore these values report that ASRE-DLP and ASRE-MMP is reliable with subscales.

FINDINGS AND DISCUSSION

Findings Related To the First Research Problem

The daily lesson plan designed by PMT is evaluated independently with using the ASRE-DLP by researchers. Findings related to the evaluation of daily lesson plan are given in Table 2.

Table 2. Evaluation of daily lesson plan according to the ASRE-DLP

Criteria	The 1st researcher	The 2nd researcher
Determination Of Preparatory Activity	3	3
Determination Of The Modeling Task	2	2
Uniqueness	3	3
Visual Design	2	2
Conducting Research	3	3
Spelling And Grammar	3	2
Determination Of The Amount Of Time	1	1
Total	**17**	**16**

In line with this scoring, PMT's success in determining of preparatory activity, uniqueness, and conducting research is found to be highly successful. It can be said that preparatory activity is sufficient in measuring readiness effective for transition to modeling task. Activities are unique and different from others. In conducting research phase, there is an extensive research is made and reflects on the content of plan. There is not any error in formal partition. Modeling task is seen as inappropriate for the 8th grade students' levels, thus his ways of determining the modeling task is found to be acceptable. Applying of visual, verbal and charming elements is found to be partially sufficient and this criterion partially facilitates the understanding of the problem. Determining the amount of time is considered as inappropriate for modeling process. As a result from the scores given by researchers, the PMT shows a moderate level of success in preparing the daily lesson plan.

Findings Related To the Second Research Problem

Modeling based teaching is observed by an observer who had a 3- month training on mathematical modeling like PMT and researchers. Observer is selected randomly for this study. Observer took field notes by using the OF-MMP. Observation data were analyzed descriptively and thematic-coding was made. Table 3 presents the finding with regard to observation of the mathematical modeling process.

Table 3: Findings related to the observation of the prospective mathematics teacher's skills in teaching practice on modeling based teaching

Themes	Sub-themes	Codes	Field notes
Preparing environment of the class	The physical structure of the class	Appropriateness for the modeling applications	The physical structure of the class was not appropriate to the study, but prospective teacher organized the class to be effective
	Having knowledge about subjects	High level of success Effective communication	PMT knew that there were many successful students in the class from observations of other prospective teachers' applications, conversations with mathematics teachers and he knew some of the students from out of class. PMT had an effective communication with students and mathematics teacher.
	Creating groups	Without help Taking student opinions Before teaching Group number Homogeneous distribution between groups Heterogeneous distribution intra-groups Time	PMT formed the groups without the assistance of the mathematics teacher at the beginning of the course. PMT had the knowledge about students and took their opinions for creating groups. Creation of groups before teaching was more appropriate but if he had taken help to create groups it would be good. 8 is a reasonable number of group decision. There was homogeneous distribution of the groups. Groups were similar to each other. There was heterogeneous distribution intra-group. Creating the groups took 5 minutes.
Preparation activity	Effects on the students	Prepared students to the modeling task Recalled the preliminary information Attracted attention Measured readiness Served to its purpose	Activity prepared students to the modeling task, recalled the preliminary information about prisms, attracted their attention and measured readiness. Preparation activity served to its purpose.
Modeling task	Modeling process	Understanding with assistance Active participation Working cooperatively Participating in class discussions Expressing ideas advocating groups' solutions Deciding to the correct solutions with discussions.	They read the question immediately and tried to figure out the problem. Groups understood the problem with the assistance of PMT. PMT made explanations to be needed. Groups dealt on the modeling task with all of the members. They had difficulties at the first time but then they got used to the application. They actively involved and worked cooperatively. Participation level was high. Groups completed the task in the given time. Students participated in class discussions, were able to express their ideas and advocated their groups' solutions. They decided to the correct solutions with discussing all of the groups' solutions.
	Mathematical context	Using mathematical representations and terminology correctly Making comparisons Using approximate values for numerals.	Groups used the mathematical representations and terminology correctly. They made comparisons solutions by using approximate values for numerals. Their drawings were accurate.

Themes	Sub-themes	Codes	Field notes
		Accurate drawings	
	Effects on students	Attracting attention Arising curiosity Reinforcing learning Inquiry- critical thinking Higher-order thinking Abstract thinking Peer learning Developing common thought Experiencing a different application. Permanent learning Effective learning Realizing mistakes	I think modeling task was effective on attracting of their attention, aroused their curiosity, amused them and reinforced learning. Inquiry- critical thinking skills, higher-order thinking and abstract thinking skills were attained. They learned from each other. Creative ideas emerged. They developed a common thought and experienced a different application. Permanent learning and effective learning took place. They realized their mistakes.

PMT started the course with asking questions about situations that students face daily in life. In this process, he asked questions such as, "There are buildings all around us, and all of them have roofs. Which geometric shapes do these roofs look like?" Students' responses were triangular prism, square pyramid, square prism, rectangular prism etc. Accordingly, he asked what the triangular prism and pyramid look like. In this context, he asked one more question that provided enrichment to the learning and supported organizing different thoughts.

PMT*: Well, you have got a block of cheese in the cubic. We want to make triangular prism with this cheese. How do you cut the cheese?*

Students: Diagonal, at the corners.

(There were signifiers with their hands; one student raised her finger to come to the board. She drew a cube, and showed the section by scanning. She said that if we cut and divided the part into two, we can obtain two triangular prisms.)

PMT*: So, what are the bases of prisms?*

Students: Triangle, square, rectangle...

PMT*: Is the base rectangular? Which geometric shapes are the sides of surfaces?*

Students: Square...

PMT*: Square? Are you sure?*

Students: Square, yea…

PMT*: I said that the cheese is in cubic shape.*

Students: We said that, sides are squares...

PMT*: I did not say it is the wrong answer... Well, what are the bases?*

Students: Triangle…

The following is understood from this conversation: He was evaluating the readiness of students, giving students' some opportunities to acquire mathematical competencies and preparing students to the modeling task. After submission of the modeling task, groups started the modeling cycle. Firstly, the problem situation has to be understood by the groups. The PMT deals primarily with groups who are asking questions and then fallows all of the group studies. He provides an effective and learner-oriented classroom and guidance; fosters students' independence and supports thought; stimulates cognitive and meta-cognitive activities and gave students' various opportunities to explain their thoughts independently. These skills are similar to the skills identified by Lesh & Doerr (2003, p. 11). In the last section of modeling cycle, teacher started the process of inquiry. He asked questions to whole class such as;

"*Which of these solutions do you think is most sensible? Which results are more convenient to real situation?", "Does this result fit the real situation?*"…etc. Teachers' modeling treatments in the classroom are described by Blum & Ferri (2009) as such: acting like a maestro while teaching a mathematical subject, ensuring cognitive activation of learners and effectively managing a learner-oriented classroom. These treatments have similarities with skills in teaching practice of the PMT in our study.

When students deal with modeling tasks, prospective mathematics teacher stimulates groups' thought with cognitive and meta-cognitive activities. For example, the dialogues between groups and PMT are given as follows:

S2: If we thought lengths of the roof are 25m and 100m, we could find an approximate value.

PMT*: Have you ever notice the number of windows for calculating the lengths?*

S3: No, we did not include windows.

After this dialogue, students turn back to the picture again and get another perspective when they engaged in cognitive and meta-cognitive activities. Students give the estimated values to the lengths according to the data given in the modeling task. Another group expresses that the train station looks like a prism and so they begin with the surfaces of prism. PMT asks some questions on what they think, in this way he is questioning their thought processes in reasoning. Their interaction is given below to illustrate this situation.

S1: $6a^2 = 2525$. Divide 2525 by 6.

PMT*: Why will divide 2525 by 6?*

S1: because this part is absent.

S2: Then, it must be 5, not 6.

They counted the surfaces of the station by thinking that the station is like a prism. The consensus was about 6. Thereupon, PMT says that the building's ground floor is 2525 square meters and wants them to think about the houses' sitting area, floor. K1 makes drawing on paper by separating the station into 3 parts and says that:

S1: So, there are 3of a^2

S2: How is there 3 of a^2?

S4: teacher is saying that, look at the sub-base of eraser.

*(*S1 is scanning the base by drawing a cube in the meantime)

S2: ok, I say the same.

The consensus was provided on the ground. Then they start to question about the data given in the problem. But this time their focus is on the geometric shape of the train station.

S1: train station is as a whole, he says, sitting in the garden with 2525 square meters, 6200 is only the total of the roofs.

S2: 2525 is a full-square? I do not understand, this figure as a whole is a rectangle or square?

Prospective mathematics teacher: What do you think it looks like?

S3: Rectangle

S1: I think, square.

S2: Then, I say that 2525 has to be square of something.

The 3rd group passes similar paths in reasoning like the former group did. A student from 3rd group says that: "there are 4 parts so we divided 2500 by 4. If we divide by 3, we'd take account the garden but we divided by 4 and we calculated all of them separately except the garden. We found 625 square meters". When the prospective mathematics teacher asked what the form of the roof is one of them says "triangle" as another student says "no, square". The form of the roof is considered as a planar shape.

In the modeling process, PMT gives students' various opportunities and encourages them to explain their thoughts independently. An effective and learner-oriented classroom management is exhibited by PMT and he supports students' independence such as a dialogue in another group study gives evidence as follows:.

S1: If we divide the whole of the roof into three parts (he is showing the burned roof), I thought that we could find the area of one of them.

S2: Ok, what do we do with two of them? (he is asking for the numeric information of 2525 square meters and 6200 square meters)

S3: Look at the area covered by this (2525 square meters), this is the entire surface area with garden. This is only for the roof (6200 square meters).

S2: Good, will we divide this by 3 (showing 6200 square meters)?

S4: Why do we do it?

S5: It cannot be divided to 3. 6 plus 2 equals 8 and 8 cannot be divided by 3e. (He is explaining the divisibility of 6200 by 3)

Students: let's think over the task…

When students are dealing with modeling tasks, PMT makes adaptive, independence-preserving interventions. These interventions are observed as suitable for the determinations of teacher role in the study of Blum & Ferri (2009). The PMT also creates an atmosphere which supports students' individual views besides his role of guiding students in the processes of developing models to solve problems instead of exhibiting solutions. He encourages whole class discussions in order to defense the models groups developed in each group. According to Schukajlow et.al.(2011) teachers are to be involved in the conditions when students' independency is kept optimal level. From the observations we made in our research, it can be said that the balance between his guidance (minimal) and (maximal) students' independence maintained during the modeling process. Marcou&Lerman(2007), asserts that student-centered teaching environments provides opportunities for student. Therefore, students in this study develop their modeling skills and creative thinking through an effective and student-centered modeling process with the help of the PMT.

Findings Related To the Third Research Problem

In the modeling based teaching, groups paraphrased the information given by the problem situation, explained their thoughts to each other, made drawings in the shape of the station and then most of them thought dividing the area into equal parts according to the information related to 2525 square meters. When they were developing their conceptual systems or models through the mathematization, they found relationships between the lengths of burned roof, area of station and coating material; they resized, quantified or made predictions. As they work with the rich contextual data, they would need to surface and communicate their mathematical ideas to clarify their thoughts and ensure the validity of their ideas. In this context they wrote symbols and made diagrams related to designed roofs. From this perspective, at the end of the observed modeling process, groups developed similar models. In the study of English(2009), students checked their interpretation and reinterpretation of problems and data sets, identified key problem factors, determined and applied quantification process to transform the data, and documented and supported their actions in various representational formats. Cognitive analysis of groups' modeling process are made according to the modeling cycle given in Figure 2 and evaluated by using ASRE-MMP. PMT' s scoring of groups' efforts with taking into consideration the criteria in modeling process is given in Table 4.

Table 4. Scores of groups according to the criteria in mathematical modeling process.

Criteria	1st group	2nd group	3rd group	4th group	5th group	6th group	7th group	8th group
Understanding	1	2	1	2	2	1	1	2
Simplifying/ structuring	1	2	1	3	2	1	2	3
Mathematising	2	2	1	3	2	1	2	3
Working mathematically	3	1	1	2	3	2	3	3
Interpreting	3	1	1	2	3	2	3	3
Validating	3	1	1	3	3	2	3	3
Exposing	3	1	1	2	3	2	3	3
Total score	16	10	7	17	18	11	17	20
Success level	**moderate**	**low**	**low**	**moderate**	**High success**	**moderate**	**moderate**	**High success**

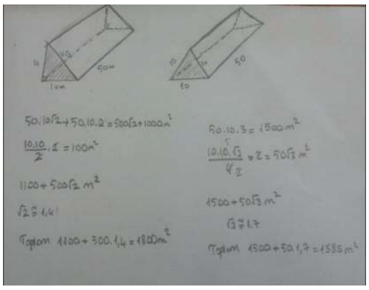
Four groups did not understand the problem situation but the other four groups understood the problem situation with the help of PMT. The fourth and eighth groups only completed the simplifying/structuring phase successfully, as they completed the previous phase with assistance. The first, third and sixth groups understand only problem situation but they could not organize and simplify the situation and couldn't associate problem situation with any mathematical idea. The second, fifth and seventh groups made the list of problem features, created a list by looking at certain features but neither they could describe the variables used in the model nor did an accurate drawing /table. Similarly the fourth and eighth groups completed the mathematization process successfully. The second, fifth and seventh groups are to be moderately successful. The first, third and sixth groups neither took into account each variable in creating model nor used mathematical representation and terminology correctly. Their model is not suitable for the problem situation.

The first, fifth, seventh and eighth worked over the mathematical problem using mathematical model and reached the correct solution. 4th and 6th groups made processing error and remaining 2 groups could not work mathematically. Almost all of the groups (75%) were unable to interpret achieved mathematical results with real outcomes in an adequate level. Accuracy of the mathematical model with appropriate data was tested, mathematical model was affirmed and the developed mathematical model is not able to be generalized for any other problems in the second and third groups. The sixth group is unable to generalize their model. The 1st,4th,5th,7th and 8th groups completed validating at high success level.

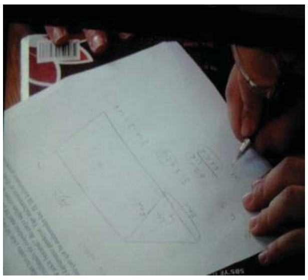
The 1st,4th,5th,6th,7th and 8th groups submitted verbal solutions of the problem and explained the solution correctly. But when different opinions were mentioned by other groups, the 2nd, 3rd, 4th and 6th groups were unable to defend their own solutions against them. The 2nd and 3rd groups could not successfully complete the last four phases. These findings are clearly pointed out in solutions.

In the exposing phase, solutions were presented on board. A student from the 8th group drew a triangular prism on the board. The floor of the station was regarded as square and total area was 2500 m^2. Lengths of the roof floors were approximately 50 m and 10 m. She drew a floor of the building that is placed on the square shaped station in order to increase the visual understandability of the shape. They used an isosceles triangle in the form of base 10m and edges 5√2m. Accordingly, the height of the triangle was 5m. Their mathematical solution was: (50x10)+(50x5√2x2)+(10x5)/2+(10x5)/2=550+500√2= 1255 square meters. In this context, required material was found to be 1255 square meters. Lengths of roof floor were regarded as 30m and 120m by the fourth group. Height of the isosceles triangle was found to be 20 m according to the right triangle (3-4-5) which is used to ease the calculation. Their mathematical solution was: (120x25x2)+(120x30)+(30x20)= 10200 square meters which was the required amount. The sixth group who designed the roof and triangular prism regarded the length of the roof floor as 150 m and 40 m. Height of the triangles were 25m, edge lengths were 40m, 5√41m and 5√41m. In this case (150x40)+(150x5√41x2)+(40x25)=6000+1500√41 square meters approximately refers to 14500 square meters of material. The roof was designed as a triangular prism by fifth group and thus a^2=2525, a=5√101m. Accordingly, their solutions and results were similar with the eighth group. The first group calculated the roof floor to be 50m and 20 m. Length of the base of the triangle was 20m, edges were 5√5m, the height was 5m. The required material was found to be (50x20)+(50x5√5x2)+(20x5)= 1100+500√2=1805 square meters. The seventh group's solution is: Right triangles with the lengths of (10m-10m-10√2m), the lengths of the roof floor were 50m and 10m. 1100+500√2 square meters (1800 square meters) material was required. In the case of an equilateral triangle (10m-10m-10m) without any changes on the roof floor, 1500+50√3 square meters (1585 square meters) material was required. According to group discussion, the eighth, fifth, first and the seventh groups' solutions are accepted to be correct solutions. Some figures are given below for illustrating the modeling process.

As a result, two groups showed a high level of success, 4 groups showed moderate level of success and 2 groups showed a low level of success according to the evaluation of ASRE- MMP.According to the results of PISA 2006, students all over the world experience problems with modeling tasks (Blum & Leiß, 2007). This situation is related to complexity structure of modeling tasks by Schukajlow et. al.(2011). In this study, students sometimes have difficulties in developing the models. This is an expected situation because of the difficulty and uncertainty of the data given in problem situation. PMT gives a sufficient time (more than the specified time in the daily lesson plan) for modeling processes and guidance. As Eric (2010) and Schukajlow et.al. (2011) pointed out, this intervention is effective on students' development of models. Students are even believed and encouraged to be successful in doing modeling tasks by prospective mathematics teacher. According to Nyman&Berry(2002), this technique-mathematical modeling- may be useful or practicability when students actively participated to the activities and ready to class discussion. However, when students forced to explain and argue their models, they uncover inaccuracies and misunderstandings. Lingefjard(2006) emphasizes a clear focus on validating process. In this study modeling process is successfully done. Students expressed that modeling task is charming and they feel enjoyment while they are learning mathematics with modeling. In this way, mathematical modeling provides students a qualified mathematics learning environment.








CONCLUSIONS

In modeling, students are presented with real world situations and are expected to use mathematics in order to rationalize these situations. Students need more mathematical understanding in order to construct valid and useful models. Mathematical modeling plays a significant role in the mathematics. Modeling, as incorporated in the curriculum recommendations of NCTM, forms the basis of classroom activities.

In this context, this study offers information about the modeling applications of students and the role of teacher. Modeling provides an effective context for developing students' problem solving skills. Moreover, modeling promises to highlight mathematical connections, addresses to the aspects of learning and reinforces students' understanding of mathematics. Modeling provides teachers an additional tool for connecting with students and motivating them. The need to study with different level students and prospective mathematics teachers aroused in order to use the research data in future researches in terms of different modeling. Modeling is difficult to teach and learn. On the other hand, by developing awareness of teacher instructional practices; students' modeling competencies can be facilitated and developed through well designed tasks with collaborative studies between researchers and PMTs in educational faculties. Finding an appropriate context within which modeling can be undertaken is not a simple task as it needs to be readily understandable and seen as relevant by students, required an appropriate level of mathematical training on modeling.

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